Closed-Form Analysis of Relay-Based Cognitive Radio Networks Over Nakagami-$m$ Fading Channels

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Abstract—We propose a general framework for a comprehensive performance analysis of cooperative spectrum sensing (CSS) in cognitive radio (CR) networks. Specifically, we investigate the detection accuracy of a relay-based CR network over independent nonidentical Nakagami-$m$ fading channels. Based on the probability density function (pdf) approach, we derive new exact and approximated closed-form expressions for the average detection probability and the average false alarm probability employing two diversity combining techniques, namely, the maximal ratio combining (MRC) scheme and the selection combining (SC) scheme. We also investigate the convergence rate of infinite series that appears in the derived exact closed-form expressions and propose to use a powerful acceleration algorithm that allows for the series termination with a finite number of terms. The results obtained reveal the importance of including the relaying link statistics and the combination techniques in the performance analysis of CR networks. The derived closed-form expression can be used to determine the energy threshold and the relaying power constraint that meet a given detection accuracy value over nonidentically distributed Nakagami-$m$ fading.

Index Terms—Cognitive radio (CR), cooperative spectrum sensing (CSS), detection probability, energy detector, false alarm probability.

I. INTRODUCTION

SIGNAL detection is a key part of spectrum sensing in cognitive radio (CR) networks. It aims to detect whether there is an unused portion of the licensed spectrum to provide opportunities for secondary users to access the available spectrum bands in the absence of licensed primary users. Therefore, reliable detection of primary users is an essential requirement for the minimization of interference to existing primary networks [1]. Unfortunately, in a real communication environment, the wireless links may be impaired due to shadowing and fading environments. Therefore, local spectrum sensing, or what is called noncooperative spectrum sensing, is unreliable and prone to error. In addition, over faded channels, the CR must have sufficiently high sensitivity to detect even extremely weak primary signals. The high cost of such highly sensitive CR terminals may limit the wide deployment of CR networks [2]. However, detection accuracy can be improved without imposing higher sensitivity requirements on the individual CR through cooperative spectrum sensing (CSS) methods. CSS adds diversity to the spectrum sensing network and helps improve the reliability of detecting the primary user [3], [4].

CSS networks achieve high diversity gain benefiting from the information exchange among the secondary users through a fusion center. It has been shown that a network of cooperative CRs, which experience different channel conditions from the target, would have a better chance of detecting the primary radio if the local observations are jointly combined at a base station [5], [6]. In addition, CSS improves the overall detection sensitivity and has been shown to reduce the detection time and increase the overall agility [7], [8].

Cooperative diversity techniques have been introduced in the literature to improve the detection accuracy in CR networks, e.g., [3] and [9]–[15]. The basic idea is to share the same spectrum band among several neighboring CRs that can relay their local sensing to help secondary users who are faraway from the primary network or to a central base station for a global decision making. In this dual-hop cooperative network, the primary user stands for the source and the common receiver stands for the destination. Clearly, there are two communication links that need to be considered to optimize spectrum sensing: first, the sensing paths between the source and the CRs; second, the relaying paths between the CRs and the common receiver.

Several cooperative diversity approaches have been proposed in the past few years. In [5], [10], [16], and [17], CSS approaches are investigated under the typical assumptions of additive white Gaussian noise (AWGN) channels. The effect of error on reporting decisions to a fusion center is considered in [3]. The study investigates the problems of CSS over independent identically distributed (i.i.d.) Rayleigh fading channels. In [9] and [18], the performance of an energy detector over wireless channels with composite multipath fading and shadowing effects is investigated. The study quantifies a single-hop CR network using the detection probability as a performance measure. Similarly, the relaying links are ignored in [19], and a single-hop network is considered. The work presents a performance analysis of energy detection over multipath fading for a system with antenna diversity reception.

Relay-based CR networks are proposed in [4], [7], [8], and [20]–[23]. A framework for performance analysis of distributed data fusion over correlated log-normal channels is introduced in [20] and extended in [23]. Based on the harmonic mean of two random variables, a novel two-step approximation method is proposed to handle shadowing correlation over diversity
branched. Distributed networks with i.i.d. Rayleigh fading channels are considered in [7], [8], and [21]. The authors propose that some of the active CRs could act as relays to help other users located faraway from the primary network, which would improve agility as a result of the cooperative process. A comprehensive framework for the analysis of dual-hop cooperative wireless systems is proposed in [4]. The authors compute the moment generating function (MGF) of the end-to-end signal-to-noise ratio (SNR) for various fading distributions. For most fading scenarios, exact formulas have been developed, whereas for some other scenarios, tight bounds are suggested.

Existing relaying techniques are classified into amplify-and-forward (AF) protocol and decode-and-forward (DF) protocol. In the AF protocol, CR users forward their raw measurements to the fusion center. While in the DF protocol, CR users perform local detections and only forward decisions to the fusion center. The latter case offers a substantial reduction in bandwidth requirements for reporting data to the fusion center. However, the AF protocol shifts the complexity from the local radios to the fusion center and offers a simple way to report raw data to the decision maker, which has been shown to improve the spectrum sensing performance [24], [25]. Moreover, the AF protocol allows the soft-fusion policy to be used, which has been approved to achieve optimal diversity, providing a higher diversity gain and better detection accuracy compared with the decision–fusion policy [7], [8], [21], [26]. However, under a limited-time assumption, the authors in [22] have shown that decision–fusion policy can outperform soft-fusion policy if a large number of users are involved in the cooperative sensing. Based on the AF gain, the relay is referred to as a CSI\textsuperscript{1}-assisted relay or as a fixed-gain relay. The former case requires the CR to be aware of the instantaneous sensing channel state to adjust its gain according to the sensed power. On the other hand, the fixed-gain relay amplifies and forwards the received signal with a constant gain and consequently results in an output signal with variable power. Although fixed-gain relaying significantly reduces the complexity of each relay, its performance in fading channels is still less explored than the CSI-aided relay and is not expected to perform well as systems with CSI-aided relays [4], [27].

Importantly, the relaying channels convey noise to the destination, which is why it is necessary to incorporate the relaying links in the performance analysis of CR networks. Specifically, the difference between a conventional single-hop system and a relay-based dual-hop system is that the noise in the single-hop system is independent of the channel statistics, whereas the noise in the dual-hop system is not [27]. Motivated by this fact, we propose a new relay-based CSS strategy that incorporates sensing links, relaying links, diversity combining techniques, and energy detection into a general framework for a comprehensive performance analysis of CR networks. We consider a general fading model in which the sensing and the relaying channels are subjected to independent not identically distributed Nakagami-m fading. The wide versatility, experimental validity, and analytical tractability of Nakagami distribution have made it a very popular fading model for performance analysis investigations in diversity schemes and cochannel interference [28]. While initial investigations in CR network’s performance have dealt with i.i.d. diversity paths, more recent works have focused on the performance analysis over nonidentical fading distributions [18], [19].

In this paper, we extend the work in [21] and provide a probability density function (pdf)-based approach to the performance analysis of a relay-based CR network over generalized fading models. The approach employs AF relaying protocol due to its better performance compared with the DF relaying protocol. Accordingly, each CR carries out local sensing within a specified sensing time period, then acts as a relay, and forwards the local sensing to the fusion center. Based on the combined inputs, the fusion center decides whether the primary network is active or not. In the next section, we present the analytical approaches of the proposed system. The results and performance evaluation are presented in Section III.

II. System Model

A centralized CR network with L active secondary users is considered. The cooperative decision is assumed to be made by a fusion center. The secondary users \( \{CR_i\}_{i=1}^L \) stand for system relays and share the same spectrum band that is originally allocated to the primary users. The secondary users operate in a fixed time-division multiple-access scheme where the sensing and transmission phases are alternating periodically. The spectrum sensing phase consists of two time slots. In the first slot, all CRs listen to the primary user signal over the shared spectrum band, and in the second slot, each CR amplifies and relays its local sensing to the fusion center. We assume that all channels experience independent not identically distributed Nakagami-\( m \) fading. We denote by \( h_s \) and \( h_r \), the channel fading parameters of the \( i \)th sensing link and the \( i \)th relaying link, respectively. We also denote by \( n_{s_i} \) and \( n_{r_i} \), the additive Gaussian noise of the sensing and the relaying channels, respectively. The noise \( n_{s_i} \)’s and \( n_{r_i} \)’s are modeled as i.i.d. zero-mean complex Gaussian random variables with variance \( N_0 \). We assume that \( h_{s_i}, h_{r_i}, n_{s_i}, \) and \( n_{r_i} \) are pairwise independent.

Every CR relay has a maximum power constraint \( P_i \). Hence, it measures the average received signal power and scales it appropriately so that the power constraint is satisfied. As in [7], we assume that each CR has access to its channel state information, i.e., CSI assisted. This is facilitated by allowing pilot symbols to be transmitted at regular intervals. Let \( x_p \) denote the signal transmitted by the primary radio and \( A_i \) denote the amplification gain of the \( i \)th CR relay. Then, \( A_i \) is selected according to the maximum power constraint as [29]

\[
A_i = \frac{P_i}{E_i + N_0}
\]

where \( E_i = E\{h_{s_i}x_p^2\} \) is the expected power of the signal received by the \( i \)th CR relay. The fusion center combines the relayed signals, compares the resultant statistic with a decision threshold, and provides the cooperative decision.

\footnote{CSI stands for channel state information.}
A. Single-Relay System

To realize the analysis of multirelay spectrum sensing, it is essential to introduce the analysis of the single-relay case. Let $y_{i}$ denote the signal received at the RF front end of the $i$th relay, then $y_{i}$ can be expressed as

$$ y_{i} = \theta h_{s_{i}} x_{p} + n_{s_{i}} \tag{2} $$

where $\theta$ denotes the primary user indicator under two hypotheses: $H_0$ for primary user absence and $H_1$ for primary user presence. If $y_{i}$ denotes the signal received at the fusion center, then

$$ y_{i} = h_{r_{i}} \left( \sqrt{A_{i}} (\theta h_{s_{i}} x_{p} + n_{s_{i}}) \right) + n_{r_{i}} $$
$$ = \theta \sqrt{A_{i}} h_{r_{i}} h_{s_{i}} x_{p} + \sqrt{A_{i}} h_{r_{i}} n_{s_{i}} + n_{r_{i}} \tag{3} $$

Since we model the signal as a random variable with known power, the energy detector is considered to be the optimal receiver to detect the presence of the signal [30]. For energy detection, the fusion center compares the power of $y_{i}$ denoted by $Y_{i}$ with a given threshold $\lambda$ to make the spectrum sensing decision. The threshold must be properly selected to detect the primary state $\theta$, where $\theta = 0$ indicates the absence of the primary signal, and $\theta = 1$ indicates the presence of the primary signal. By definition, a false alarm occurs when the fusion center claims the primary user activity under $H_0$, whereas an accurate detection occurs when the primary user activity is claimed under $H_1$. Hence, the false alarm probability ($P_{fa}$) and the detection probability ($P_{d}$) can be evaluated by $P\{Y_{i} > \lambda | H_0\}$ and $P\{Y_{i} > \lambda | H_1\}$, respectively.

In wireless literature, e.g., [5], [7], [16], and [17], the probability of false alarm is assumed to be independent of the channel statistics. However, this is only true for single-hop communications. Specifically, the noise in the dual-hop system cannot be independent of the channel statistics since the second-hop link conveys noise to the destination, which can be easily verified through the second term in (3). Accordingly, the propagation characteristics of the second hop must be considered when computing the false alarm probability and the detection probability.

To accommodate both cases $H_0$ and $H_1$, we define $g_{i} = |h_{r_{i}}|^2$ as the instantaneous channel gain of the $i$th relaying link and $\bar{g}_{i} \triangleq E\{|h_{r_{i}}|^2\}$ as the expected value of $g_{i}$. Since $h_{r_{i}}$ follows a Nakagami-\textit{m} distribution, it is easy to verify that $g_{i}$ follows a gamma distribution given by

$$ f_{G_{i}}(g) = \left( \frac{m_{i}}{g_{i}} \right)^{m_{i}} g_{i}^{m_{i}-1} e^{-\frac{m_{i}}{g_{i}}} g, \quad g \geq 0 \tag{4} $$

Now, from (3), the mean value of $Y_{i}$ for a given $g_{i}$ can be expressed as

$$ E\{Y_{i}|g\} = \left\{ \begin{array}{ll} \sigma_{Y_{i_{0}}} = N_{0}(1 + A_{i} g), & H_0 \\ \sigma_{Y_{i_{1}}} = N_{0}(1 + (1 + \gamma_{r_{i}}) A_{i} g), & H_1 \end{array} \right. \tag{5} $$

where $\gamma_{r_{i}} = E_{i}/N_{0}$ is the average SNR at the input of the $i$th CR receiver. Accordingly, the probability of the false alarm for a given $g$ can be evaluated as

$$ P_{f_{i}|g} = P\{Y_{i} > \lambda | H_0, g\} = \int_{\lambda}^{\infty} \left( \frac{m_{i}}{\sigma_{Y_{i_{0}}}} \right)^{m_{i}} g^{m_{i}-1} \Gamma(m_{i}) e^{-\left( \frac{m_{i}}{\sigma_{Y_{i_{0}}}} \right) g} dy $$
$$ = \frac{1}{\Gamma(m_{i})} \Gamma \left( m_{i}, \frac{m_{i} \lambda}{N_{0}(1 + A_{i} g)} \right) \tag{6} $$

where the integral in (6) is evaluated with the help of [31, eq. 3.383.5]. Now, we remove the condition on $g$ and compute an average false alarm probability $P_{fb}$ by integrating over the pdf of the channel gain given in (4) as follows:

$$ P_{f_{i}} = \int_{0}^{\infty} P\{Y_{i} > \lambda | H_0, g\} f_{G_{i}}(g) \, dg $$
$$ = \int_{0}^{\infty} \frac{1}{\Gamma(m_{i})} \Gamma \left( m_{i}, \frac{m_{i} \lambda}{N_{0}(1 + A_{i} g)} \right) f_{G_{i}}(g) \, dg. \tag{7} $$

Evaluating the integral in (7) as described in Appendix A, $P_{f_{i}}$ can be mathematically expressed as

$$ P_{f_{i}} = \left( \frac{\beta_{i}}{A_{i}} \right)^{m_{i}} \sum_{q=0}^{m_{i}-1} \frac{1}{q!} \left( \frac{m_{i} \lambda}{N_{0}} \right)^{q} \sum_{n=0}^{\infty} (-1)^{n} b_{n} $$
$$ \times U \left( m_{i}; m_{i} + 1 - q - n; \frac{\beta_{i}}{A_{i}} \right) \tag{8} $$

where $\beta_{i} = m_{i}/\bar{g}_{i}$, $b_{n} = (1/n!)(m_{i} \lambda/N_{0})^{n}$, and $U(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function of the second kind defined in [32, eq. 13.1.3]. Similarly, the average detection probability $P_{d_{i}}$ can be obtained as

$$ P_{d_{i}} = \left( \frac{\beta_{i}}{1 + \gamma_{r_{i}} A_{i}} \right)^{m_{i}} \sum_{q=0}^{m_{i}-1} \frac{1}{q!} \left( \frac{m_{i} \lambda}{N_{0}} \right)^{q} \sum_{n=0}^{\infty} (-1)^{n} b_{n} $$
$$ \times U \left( m_{i}; m_{i} + 1 - q - n; \frac{\beta_{i}}{1 + \gamma_{r_{i}} A_{i}} \right). \tag{9} $$

B. Multirelay CSS System

1) SC Scheme: In the performance analysis of cooperative diversity techniques, the statistic of the maximum of a set of random variables is often important [33]. To identify this statistic, we use the selection combining (SC) technique to allow the fusion center to select the relaying link that has the highest gain among all the diversity branches. It is worth noting that the SC technique can be implemented in two different strategies [34]. In the first strategy, the combiner selects the relaying branch with the highest SNR, i.e., $\gamma_{r_{i}}$, whereas in the second strategy, the relays with the highest $\min(\gamma_{s_{i}}, \gamma_{r_{i}})$ is selected. In our CSS model, we employed the first strategy. However, instead of the SNR, the channel gain will be used to accommodate the two hypotheses, i.e., $H_0$ and $H_1$. Therefore, for $L$ inputs, the output of the SC receiver is expressed as $g_{\text{max}} = \max(g_{i_{1}}, g_{i_{2}}, \ldots, g_{i_{L}})$. 

If $Y$ denotes the signal power at the output of the combiner, then the mean value of $Y$ for a given $g$ can be expressed as

$$E\{Y|g\} = \left\{\sigma Y_0 = N_0(1 + ASGCg), \sigma Y_1 = N_0(1 + (1 + \tau_{ASC})ASGCg)\right\}, \quad H_0 \quad \text{and} \quad H_1$$

(10)

where $ASGC \in \{A_i\}_{i=1}^L$ and $\tau_{ASC} \in \{\tau_i\}_{i=1}^L$ denote the amplification factor and the average SNR associated with the selected relay, respectively. The average false alarm probability $P_{f_{SC}}$ is evaluated by averaging over the pdf of $g_{\text{max}}$ as follows:

$$P_{f_{SC}} = \frac{\int_{0}^{\infty} \int_{0}^{\infty} \gamma (m_{SC}, N_0(1 + ASGCg)) f_{g_{\text{max}}}(g) dg}{\Gamma(m_{SC},N_0(1 + ASGCg))}$$

(11)

where $m_{SC} \in \{m_j\}_{j=1}^L$ denotes the Nakagami-$m$ parameter of the link associated with the selected relay. With the help of Appendix B, $P_{f_{SC}}$ can be mathematically expressed as follows:

$$P_{f_{SC}} = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \frac{\beta_{\nu_k}^{m_k}}{\Gamma(m_k)} \frac{L}{L} \ldots \frac{m_{m_k-1}}{m_{m_k-1}} \frac{\sum_{l_k=0}^{m_{m_k-1}} \left(\prod_{i=1}^{k} \frac{\beta_{\nu_k}^{m_k}}{l_i!} \right) \sum_{q=0}^{m_{m_k-1}} \frac{1}{l_i!} \left(\frac{m_{SC}\lambda}{N_0}\right)^q \Gamma(\nu_k) A_{m_k}^{\nu_k}}{A_{m_k}^{\nu_k}}$$

$$\times \sum_{n=0}^{\infty} (-1)^n b_n U \left(\nu_k; \nu_k + 1 - q - n; \mu_k \right)$$

(12)

where $b_n = (1/n!) \left(m_{SC}\lambda/N_0\right)^n$, $\nu_k = \sum_{i=1}^{k} l_i + m_j$, and $\mu_k = \sum_{i=1}^{k} \beta_{\nu_k} + \beta_{\nu_k}$. Similarly

$$P_{d_{SC}} = \frac{\int_{0}^{\infty} \int_{0}^{\infty} \gamma (m_{SC}, N_0(1 + ASGCg)) f_{g_{\text{max}}}(g) dg}{\Gamma(m_{SC},N_0(1 + ASGCg))}$$

(11)

where $m_{SC} \in \{m_j\}_{j=1}^L$ denotes the Nakagami-$m$ parameter of the link associated with the selected relay. With the help of Appendix B, $P_{d_{SC}}$ can be mathematically expressed as follows:

$$P_{d_{SC}} = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \frac{\beta_{\nu_k}^{m_k}}{\Gamma(m_k)} \frac{L}{L} \ldots \frac{m_{m_k-1}}{m_{m_k-1}} \frac{\sum_{l_k=0}^{m_{m_k-1}} \left(\prod_{i=1}^{k} \frac{\beta_{\nu_k}^{m_k}}{l_i!} \right) \sum_{q=0}^{m_{m_k-1}} \frac{1}{l_i!} \left(\frac{m_{SC}\lambda}{N_0}\right)^q \Gamma(\nu_k) A_{m_k}^{\nu_k}}{A_{m_k}^{\nu_k}}$$

$$\times \sum_{n=0}^{\infty} (-1)^n b_n U \left(\nu_k; \nu_k + 1 - q - n; \mu_k \right)$$

(12)

In addition, over i.i.d. Nakagami-$m$ channels $P_{f_{SC}}$ and $P_{d_{SC}}$ are expressed as follows:

$$P_{f_{SC}} = \frac{\beta_{\nu_k}^{m_k}}{\Gamma(m_k)} \frac{L}{L} \ldots \frac{m_{m_k-1}}{m_{m_k-1}} \frac{\sum_{l_k=0}^{m_{m_k-1}} \left(\prod_{i=1}^{k} \frac{\beta_{\nu_k}^{m_k}}{l_i!} \right) \sum_{q=0}^{m_{m_k-1}} \frac{1}{l_i!} \left(\frac{m_{SC}\lambda}{N_0}\right)^q \Gamma(\nu_k) A_{m_k}^{\nu_k}}{A_{m_k}^{\nu_k}}$$

$$\times \sum_{n=0}^{\infty} (-1)^n b_n U \left(\nu_k; \nu_k + 1 - q - n; \mu_k \right)$$

(14)

$$P_{d_{SC}} = \frac{\beta_{\nu_k}^{m_k}}{\Gamma(m_k)} \frac{L}{L} \ldots \frac{m_{m_k-1}}{m_{m_k-1}} \frac{\sum_{l_k=0}^{m_{m_k-1}} \left(\prod_{i=1}^{k} \frac{\beta_{\nu_k}^{m_k}}{l_i!} \right) \sum_{q=0}^{m_{m_k-1}} \frac{1}{l_i!} \left(\frac{m_{SC}\lambda}{N_0}\right)^q \Gamma(\nu_k) A_{m_k}^{\nu_k}}{A_{m_k}^{\nu_k}}$$

$$\times \sum_{n=0}^{\infty} (-1)^n b_n U \left(\nu_k; \nu_k + 1 - q - n; \mu_k \right)$$

(15)

In this case, it is important to mention that $\nu_k = \sum_{i=1}^{k} l_i + m_j$ and $\mu_k = (k + 1)\beta$.

2) MRC Scheme: Many performance analysis problems require determination of statistics of the sum of the squared envelopes of the faded signals over several diversity paths, which can be achieved through maximal ratio combining (MRC) technique. MRC receiver weights its input signals with respect to their channel statistics and is known to be of high performance [33]. For $L$ inputs, the output of the MRC receiver is given by

$$y = \sum_{i=1}^{L} \sqrt{A_i} h_{r_i} (\theta h_{s_i} x_p + n_{s_i}) + n_r,$$

$$= \theta \sum_{i=1}^{L} \sqrt{A_i} h_{r_i} h_{s_i} x_p + \sum_{i=1}^{L} \sqrt{A_i} h_{r_i} n_{s_i} + n_r.$$  (16)

In a similar way to the SC scheme analysis, if $Y$ denotes the power at the output of the MRC receiver, then for given $g_i$’s, the mean value of $Y$ is given by

$$E\{Y|g_i\} = \left\{\sigma Y_0 = N_0 \left(1 + \sum_{i=1}^{L} A_i g_i\right), \sigma Y_1 = N_0 \left(1 + \sum_{i=1}^{L} (1 + \tau_{ASC}) A_i g_i\right), H_0 \quad \text{and} \quad H_1 \right\} \text{,}$$

(17)

Therefore, the average false alarm probability for the MRC case is given by

$$P_{f_{MRC}} = P\{Y \geq \lambda | H_0\} = \int_{0}^{\infty} \cdots \int_{0}^{\infty} f(g_1|H_0) \cdots f(g_L|H_0) \cdot \left(f(g_1 \cdots dg_L) \right).$$

(18)
The expression of $P_{f_{\text{farc}}}^c$ in (18) cannot straightforwardly
be evaluated. However, we can rewrite this expression as the
expectation over $g_i$'s such that

$$P_{f_{\text{farc}}}^c = E_{g_1, \ldots, g_L} \{P\{Y \geq \lambda | H_0, g_1, \ldots, g_L\}\} = E_{g_1, \ldots, g_L} \left\{ \frac{1}{\Gamma(m_j)} \left( \frac{m_j \lambda}{N_0 (1 + \sum_{i=1}^L A_i g_i)} \right)^{m_j} \cdot f(g_1 | H_0) \cdots f(g_L | H_0) \, dg_1 \cdots dg_L. \right\} \quad (20)$$

where $f_R(r)$ is the pdf of random variable $R$. This integral can be
evaluated as described in Appendix C, yielding

$$P_{f_{\text{farc}}}^c = \int_0^\infty \frac{1}{\Gamma(m_j)} \left( \frac{m_j \lambda}{N_0 (1 + r)} \right)^{m_j} \cdot f_R(r) \, dr \quad (21)$$

To simplify the computations of the previous expectation, we
define the following random variable:

$$R = \sum_{i=1}^L A_i g_i. \quad (20)$$

Since each $g_i$ follows gamma distribution, random variable $R$
is a sum of mutually independent gamma variates. Thus, the
expression of $P_{f_{\text{farc}}}^c$ in (19) can be expressed as

$$P_{f_{\text{farc}}}^c = \left[ \sum_{j=1}^L \left( \frac{\beta_j}{A_j} \right)^{m_j} \sum_{j=1}^L \sum_{v=1}^{m_j} \frac{1}{q!} \left( \frac{m_j \lambda}{N_0} \right) q \sum_{n=0}^\infty (-1)^n b_n U(v; v + 1 - q - n; \frac{\beta_j}{A_j}) \right]. \quad (22)$$

Similarly, the average detection probability can be expressed as

$$P_{d_{\text{farc}}}^c = \left[ \sum_{j=1}^L \left( \frac{\beta_j}{A_j} \right)^{m_j} \sum_{j=1}^L \sum_{v=1}^{m_j} \frac{1}{q!} \left( \frac{m_j \lambda}{N_0} \right) q \sum_{n=0}^\infty (-1)^n b_n \right] \times U(\frac{\beta_j}{(1 + \gamma_s) A_j}). \quad (23)$$

where

$$b_{jv} = \sum_{j=1}^{m_j-1} \binom{m_j-v-1}{j_1} B_{j_m-v-1-j_1}^{m_j-1-j_1} \sum_{j=0}^{j_1-1} \binom{j_1-1}{j} \Delta_j \left( \frac{m_j - v}{m_j} \right)! \times B_j^{j_1-j_2} \sum_{j=0}^{j_2-1} \binom{j_2-1}{j_1} B_j^{j_2-j_1-j_2} \cdots \frac{\Delta_j}{(m_j - v)!}$$

with

$$\Delta_j = \prod_{i=1}^L \left( \frac{\beta_i}{A_i} - \frac{\beta_j}{A_j} \right)^{m_i}$$

$$B_j^r = (-1)^{r+1} \left[ \sum_{i=1}^L \frac{\beta_i}{A_i} - \frac{\beta_j}{A_j} \right]^{-(t+1)}.$$
The algorithm repeatedly applies the recursive expression in (27) to estimate the converging point of \( n \) while constantly adding adequate terms of \( \sum_{n=0}^{\infty} (-1)^n b_n \) to reach the required accuracy. The sequence \( (\varepsilon^n_i) \) is called the \( l \)-th column, and its construction can be graphically represented, as shown in Fig. 1. The upsweeping diagonals converge very quickly to the limit. This diagonal sequence (circled in Fig. 1) is the result of the \( \varepsilon \)-algorithm and is called the accelerated sequence.

**D. Approximation Analysis**

Further to the acceleration algorithm, we propose a new computational approach to derive the average false alarm probability and the average detection probability. If \( A_i \) is assumed to be high enough, then an approximated expansion of the incomplete gamma function can be used as follows: For a large \( A_i \), we have \((m + \frac{1}{\gamma_i_s}A_{g_i}) \gg 1\), and \( \Gamma(m_i, (m_i/ (N_0(1 + \frac{1}{\gamma_i_s}A_{g_i}))) \) is approximated to \( \Gamma(m_i, (m_i/ (N_0(1 + \tau_{\gamma_i_s})A_{g_i}))) \), which can be expanded as \((m_i - 1)!e^{(m_i - 1)}(N_0(1 + \frac{1}{\gamma_i_s})A_{g_i}) \). This approximation alleviates the complexity of using the pdf approach in deriving \( P_f \) and \( P_d \), as will be shown through the derivation of the approximated closed-form expressions.

Since the average detection probability is evaluated by averaging over the pdf of \( g_{max} \), then for \((1 + \frac{1}{\tau_{\gamma_i_s}})A_{g_s} \gg 1\), we have

\[
\mathcal{P}_{d_{sc}} = \frac{1}{\Gamma(m_i)} \int_0^\infty \Gamma(m_i, (m_i/ (N_0(1 + \frac{1}{\gamma_i_s})A_{g_s}))) f_{g_{max}}(g) dg.
\]

Substituting for \( f_{g_{max}}(g) \) from (41) (see Appendix B) and using the series summation \( \Gamma(m, x) = (m - 1)!e^{-x} \sum_{n=0}^{m-1} (x^n/n!) \), the expression in (28) becomes

\[
\mathcal{P}_{d_{sc}} = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \sum_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} \sum_{n_i=1}^{L} \frac{\beta^m_j}{n_i} \sum_{l_0=1}^{m_{n_i}-1} \prod_{l_k=0}^{L-1} \frac{1}{l_k!} \sum_{q=0}^{m_{n_i}-1} \frac{1}{q!} \left( \frac{m_{SC}}{N_0(1 + \tau_{\gamma_i_s})A_{SC}} \right)^{q} \times \int_0^\infty g_{\nu_k,j}^{-\nu_q - 1} e^{-(\beta/\gamma_i_s)g} dg.
\]

Using \( \int_0^\infty x^{\nu_q} e^{-(\beta/\gamma_i_s)g} dx = 2(\beta/\gamma_i_s)^{v/2} \) \( K_{\nu}(2\sqrt{\beta/\gamma_i_s}) \) [31, eq. 3.471.9] to evaluate the integral in (29) yields

\[
\mathcal{P}_{d_{sc}} = \frac{2 \beta^m}{\Gamma(m)} \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \sum_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} \sum_{n_i=1}^{L} \frac{\beta^m_j}{n_i} \sum_{l_0=1}^{m_{n_i}-1} \prod_{l_k=0}^{L-1} \frac{1}{l_k!} \sum_{q=0}^{m_{n_i}-1} \frac{1}{q!} \left( \frac{m_{SC}}{N_0(1 + \tau_{\gamma_i_s})A_{SC}} \right)^{q} \times \frac{\nu_k + q}{\nu_k - q} K_{\nu_k,j} \left( 2 \sqrt{\frac{m_{SC}e_k}{N_0(1 + \tau_{\gamma_i_s})A_{SC}}} \right).
\]

where \( K_{\nu}(\cdot) \) is the \( \nu \)-order modified Bessel function of the second kind defined in [32, eq. 9.6.22]. It is easily shown that expression (30) does not have the infinite-series term, and therefore, it provides a simple computational method to compute the average detection probability. Similarly, for i.i.d. Nakagami-\( m \) paths, we have

\[
\mathcal{P}_{d_{m_{Nak}}} = \left[ \prod_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} \right] \sum_{j=1}^{L} \sum_{v=1}^{m_j} \frac{(-1)^v b_{jv}}{\Gamma(v)} \times \sum_{q=0}^{m_j-1} \frac{1}{q!} \left( \frac{m_j}{N_0} \right)^{q + \frac{1}{2}} \left( \frac{1 + \frac{1}{\gamma_i_s}A_j}{\beta_j} \right)^{q + \frac{1}{2}} \times \frac{1}{\mu_k} \times \nu_k - q K_{\nu_k,j} \left( 2 \sqrt{\frac{m_{SC}e_k}{N_0(1 + \tau_{\gamma_i_s})A_{SC}}} \right).
\]

Similarly, the average detection probability for the MRC sch-
and for i.i.d. Nakagami-\(m\) paths, we have
\[
\mathcal{P}_{d_{\text{MRC}}} = \frac{2\beta L m}{((1 + \gamma_s) A)^L m \Gamma(L m)} \sum_{q=0}^{m-1} \frac{1}{q!} \left( \frac{m \lambda}{N_0} \right)^{\frac{L m - q}{2}}
\]
\[
\times \left(\frac{1}{\beta}\right)^{L m - q} \frac{K_{L m - q}}{2\sqrt{\frac{m \beta \lambda}{N_0(1 + \gamma_s) A}}}.
\]  
(33)

The same approximation approach can be also used to derive the approximated \(\mathcal{P}_{f_{\text{SC}}}\) and \(\mathcal{P}_{f_{\text{MRC}}}\) but under the assumption \(A_i g_i \gg 1\).

To validate the accuracy of the approximation, we define
\[
I_k = \sum_{q=0}^{m-1} \frac{1}{q!} \left( \frac{m \lambda}{N_0} \right)^q \int_0^\infty (1 + (1 + \gamma_s) A g)^{-q} g^{\nu-1}
\]
\[
\times e^{-\frac{m \lambda}{N_0(1 + \gamma_s) A}} g \mu^{-\nu} e^{-\mu g} dg.
\]  
(34)

For \((1 + \gamma_s) A g \gg 1\), \(I_k\) in (34) can be approximated as
\[
I_k \approx \sum_{q=0}^{m-1} \frac{1}{q!} \left( \frac{m \lambda}{N_0(1 + \gamma_s) A} \right)^q \int_0^\infty g^{\nu-1} e^{-\frac{m \lambda}{N_0(1 + \gamma_s) A}} g \mu^{-\nu} e^{-\mu g} dg.
\]  
(35)

Using [31, eq. 3.471.9], the previous integral is evaluated as follows:
\[
I_k \approx 2 \sum_{q=0}^{m-1} \frac{1}{q!} \left( \frac{m \lambda}{N_0(1 + \gamma_s) A} \right)^q \frac{1}{(\nu-q)}^{\frac{\nu-q}{2}}
\]
\[
\times K_{\nu-q} \left( 2 \sqrt{\frac{m \lambda \mu}{N_0(1 + \gamma_s) A}} \right).
\]  
(36)

It is shown from the approximated expressions derived for \(\mathcal{P}_d\) in (30)–(33) that all the finite sums can be calculated exactly and \(I_k\) is the only approximated term. Therefore, validating the computation accuracy of \(I_k\) inevitably validates the accuracy of these expressions. In other words, the accuracy of the average detection probability depends on the accuracy of \(I_k\). For instance, a four-decimal-point accuracy is achieved in the average detection probability when four-decimal-point accuracy is achieved in \(I_k\). In fact, validating \(I_k\) offers a simple mean to avoid performing multiple simulation tests to validate each of the derived approximated expressions individually. In Section III, the accuracy of this approximation is numerically verified by simulation results generated through a Monte Carlo test.

### III. Performance Evaluation

To validate the accuracy of the derived closed-form expressions, we assume that \(L\) secondary users are deployed in a centralized CR network. The sensing and the relaying channels are assumed to be subjected to independent not identically distributed Nakagami-\(m\) fading. An upper bound of \(\mathcal{P}_f < 0.1\) is considered for the selection of decision threshold. This upper bound is recommended in the literature and the IEEE 802.22 Standard [36]. Noise variance \(N_0\) is set to a unity (0 dB). The derived closed-form expressions of \(\mathcal{P}_f\) and \(\mathcal{P}_d\) are computed with Mathematica-8 software package [37].

To determine the number of terms \(N\) required to evaluate the infinite-series expressions derived for \(\mathcal{P}_d\), we list in Table I the minimum number of terms required to evaluate the expressions given in (13) and (23) for different values of \(\lambda\). As shown, the infinite series converges with fewer number of terms for small \(\lambda\)’s, and an accuracy value up to four decimal points is obtained. However, for large \(\lambda\)’s, a large number of sum-up terms are required to evaluate \(\mathcal{P}_d\) using the exact expressions in (13) and (23). Therefore, it is more appropriate to use the approximated expressions in (30) and (32) due to their computational simplicity.

Using a large number of terms to evaluate the infinite series, i.e., \(\sum_{n=0}^{\infty} (-1)^n b_n\), significantly slows down the convergence rate. Therefore, when \(\lambda\) becomes large, it is necessary to use the \(\varepsilon\)-algorithm to improve the convergence rate. Table II shows the number of terms required to evaluate \(\mathcal{P}_d\) with an accuracy value of up to four decimal points using the \(\varepsilon\)-algorithm to accelerate (13) and (23). We observed that the use of \(\varepsilon\)-algorithm dramatically reduces the points of convergence and highly improves the convergence rate.

To validate the accuracy of the proposed approximation, we plot in Fig. 2 \(I_k\) versus \(\lambda\) for \(P = 0, 5, 10,\) and 20 dB. Analytical results obtained from the \(I_k\) definition given in (36) are compared with Monte Carlo simulations generated over 100 000 iterations to evaluate the integral-form expression given in (34). Obviously, for large \(\lambda\)’s, the analytical results

<table>
<thead>
<tr>
<th>Table I</th>
<th>Converging Point Required to Evaluate (\mathcal{P}_d) for SC and MRC Schemes, (L = 3), and (m = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC Scheme, Equation (13)</td>
<td>9 23 42 69 102</td>
</tr>
<tr>
<td>MRC Scheme, Equation (23)</td>
<td>5 16 32 50 69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II</th>
<th>Converging Point Required to Evaluate (13) and (23) With (\varepsilon)-Algorithm, (L = 3), and (m = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC Scheme, Equation (13)</td>
<td>8 12 16 19 22</td>
</tr>
<tr>
<td>MRC Scheme, Equation (23)</td>
<td>3 7 10 13 16</td>
</tr>
</tbody>
</table>
match well with the simulation results, and an accuracy value of four decimal points is achieved for $\lambda > 15$ with all the selected scenarios for the power constraint. For small $\lambda$’s, the few number of terms required to evaluate $\sum_{n=0}^{\infty} (-1)^n b_n$ makes the exact expressions more appropriate to compute $P_f$ and $P_d$ due to their high accuracy. It is also observed that the analytical and the simulation curves become closer to each other as the power constraint increases, which comes from the fact that the errors resulting from approximating $N_0(1 + (1 + \tau_s)A_i)$ into $N_0(1 + \tau_s)A_i$ decrease as $P$ increases.

In Fig. 3, we compare the analytical results obtained for $P_d$ using the exact expressions in (13) and (23) and the approximated expressions in (30) and (32) to Monte Carlo simulation results obtained over 100,000 iterations. Generally, the analytical results match well with the simulated results for both schemes over a wide range of the decision threshold $\lambda$. Compared with the MRC scheme, the approximated curve of the SC scheme shows a wider mismatch with the simulated curve at small $\lambda$’s, which is expected as the approximation errors in the case of the MRC scheme are less than those of the SC scheme.

In Fig. 4, we describe the receiver performance through its complementary receiver operating characteristic (ROC) curves. The performance of the proposed AF approach is evaluated using the expression given in (23) and compared with a DF approach to different channel conditions. For the DF approach, a false alarm probability and a detection probability are computed at each CR user by $P_{f,i} = \Gamma(m_i/(m_i\lambda_i/N_0))/\Gamma(m_i)$ and $P_{d,i} = \Gamma(m_i/(m_i\lambda_i/N_0(1 + \tau_s))) / \Gamma(m_i)$, respectively, where $\lambda_i$ is the local decision threshold. The cooperative decision then computed at the fusion center using a logic-based OR rule. A similar approach is introduced in [24]. The figure shows that the AF approach outperforms the DF approach to the selected values of fading parameter $m$. The AWGN case is also plotted for comparison purposes. AWGN channels are frequently assumed in literature to investigate the energy detection performance [5], [10], [17], [22]. Clearly, such an assumption overestimates the detection accuracy, as shown in the plotted curves.

In Fig. 5, we plot the detection probability of the SC scheme (dot-line curves) and the MRC scheme (solid-line curves) for different values of the sensing channel average SNR, i.e., $\tau_s$. The average detection probabilities of the SC and MRC schemes are computed using the exact expressions given in (13) and (23), respectively. There is an obvious improvement in the performance of both combining schemes with each step of 5-dB increase in $\tau_s$ from -10 to 10 dB. The figure shows that the MRC scheme outperforms the SC scheme for all the selected values of $\tau_s$. For low SNR applications, it is more appropriate to use the MRC scheme due to the significant difference in its performance compared with the SC scheme, as shown in the plotted curves.

Fig. 6 shows that detection accuracy is significantly improved by increasing power constraint $P$. The expressions given in (13) and (23) are used to evaluate $P_d$ for different values of fading parameter $m$. For $m = 6$, a value of $P_d > 0.9$ is achieved when $P$ increases to 8 dB. This matches the target value of $P_d = 0.9$, which is frequently used in literature as a lower bound to the detection probability [36]. This target value is also achieved for other $m$ scenarios but with higher power constraints. The high performance achieved with higher values of the fading parameter $m$ refers to the fading severity inversely proportional with $m$. Furthermore, the MRC scheme shows a better performance compared with the SC scheme, specifically at the low-power region. Roughly, a gain of 3 dB is achieved within the range 3–10 dB of the power constraint for $m = 3$ when the combiner is switched from the SC scheme to the MRC scheme, yet the SC performance becomes very close to
impaired with Nakagami-m fading. Through comprehensive evaluations, we show the importance of including the relaying links and the combining techniques into the performance analysis of CR networks. In addition, we validate the computational accuracy of the derived closed-form expressions through Monte Carlo simulation results. This paper investigates the way the detection accuracy varies with the number of diversity branches, the fading severity, and the power constraint. With the MRC receiver, a gain of up to 8 dB is observed when four cooperative users are engaged. The most interesting observation is that there is no need to increase the relaying power if a target detection probability is achieved since the improvements gained beyond that are not significant.

**APPENDIX A**

**DERIVATION OF \( \overline{P}_{f_{i}} \) FOR SINGLE-RELAY SPECTRUM SENSING**

Substituting \( f_{G_{i}}(g) \) from (4) into (7) yields

\[
\overline{P}_{f_{i}} = \left( \frac{m_{i}}{g_{i}} \right)^{m_{i}} \frac{1}{\Gamma(m_{i})} \int_{0}^{\infty} \frac{1}{\Gamma(m_{i})} \Gamma \left( m_{i}, \frac{m_{i} \lambda}{N_{0}(1 + A_{i}g)} \right) \times g^{m_{i} - 1} e^{-\frac{m_{i} \lambda}{N_{0}(1 + A_{i}g)}} dg. \tag{37}
\]

Using the series summation \( \Gamma(m, x) = (m - 1)!e^{-x}\sum_{n=0}^{m-1}(x^{n}/n!) \) [31, eq. 8.352.2] with the fact that \((m - 1)! = \Gamma(m)\), (37) becomes

\[
\overline{P}_{f_{i}} = \left( \frac{m_{i}}{g_{i}} \right)^{m_{i}} \frac{1}{\Gamma(m_{i})} \sum_{q=0}^{m_{i} - 1} \frac{1}{q!} \left( \frac{m_{i} \lambda}{N_{0}} \right)^{q} \int_{0}^{\infty} (1 + A_{i}g)^{-q} \times g^{m_{i} - 1} e^{-\frac{m_{i} \lambda}{N_{0}(1 + A_{i}g)}} dg. \tag{38}
\]

Using \( e^{-(a/b+x)} = \sum_{k=0}^{\infty} (-1)^{k} a^{k}/k!(b + x)^{k} \), we have

\[
\overline{P}_{f_{i}} = \left( \frac{m_{i}}{g_{i}} \right)^{m_{i}} \frac{1}{\Gamma(m_{i})} \sum_{q=0}^{m_{i} - 1} \frac{1}{q!} \left( \frac{m_{i} \lambda}{N_{0}} \right)^{q} \sum_{n=0}^{\infty} (-1)^{n} \frac{1}{n!} \times \left( \frac{m_{i} \lambda}{N_{0}} \right)^{n} \int_{0}^{\infty} (1 + A_{i}g)^{-(q+n)} g^{m_{i} - 1} e^{-\frac{m_{i} \lambda}{N_{0}(1 + A_{i}g)}} dg. \tag{39}
\]

The previous integral is evaluated with the aid of

\[
\int_{0}^{\infty} e^{-px} x^{q-1}(1 + ax)^{−v} dx = \Gamma(q)/a^{q}U(q; q + 1 − v; p/a)) \]

[31, eq. 3.383.5] to yield the result shown in (8).

**APPENDIX B**

**DERIVATION OF \( \overline{P}_{f_{SC}} \)**

Under nonidentical fading, the general form for the pdf \( f_{G_{\text{max}}}(g) \) is given by [38, 39]

\[
f_{G_{\text{max}}}(g) = \sum_{j=1}^{L} f_{G_{j}}(g) \prod_{i=1 \atop i \neq j}^{L} F_{G_{i}}(g) \tag{40}
\]
where $F(.)$ denotes the cumulative distribution function. The previous pdf is evaluated for integer values of $m$ in [40] as follows:

\[
G_{\text{max}}(g) = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \sum_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} L \ldots \sum_{l_0=0}^{m_{n_k-1}} \sum_{i_1=0}^{l_1=0} \frac{e^{-\mu_k g}}{\mu_k g}.
\]

Using (41), the average false alarm probability can be expressed as follows:

\[
\mathcal{P}_{\text{FSC}} = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \sum_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} L \ldots \sum_{l_0=0}^{m_{n_k-1}} \sum_{i_1=0}^{l_1=0} \frac{e^{-\mu_k g}}{\mu_k g} \times \Gamma\left(m_{SC}, \frac{m_{SC} \lambda}{N_0(1 + A_{SC} g)} \right) g^{\nu_k s-1} e^{-\mu_k g}.
\]

Using the series summation $\Gamma(m, x) = (m - 1)! e^{-x} \left(\sum_{n=0}^{m-1} \frac{x^n}{n!}\right)$ and the fact that $e^{-a(x+b)} = \sum_{k=0}^{\infty} \frac{(a)^{k}}{k!} (x + b)^k$ in a way similar to Appendix A, we have

\[
\mathcal{P}_{\text{FSC}} = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \sum_{j=1}^{L} \frac{\beta^m_j}{\Gamma(m_j)} L \ldots \sum_{l_0=0}^{m_{n_k-1}} \sum_{i_1=0}^{l_1=0} \frac{e^{-\mu_k g}}{\mu_k g} \times \Gamma\left(m_{SC}, \frac{m_{SC} \lambda}{N_0(1 + A_{SC} g)} \right) g^{\nu_k s-1} e^{-\mu_k g}.
\]

Following the same steps in Appendixes A and B, to evaluate the previous integral, (47) becomes

\[
\mathcal{P}_{\text{FSC}} = \prod_{j=1}^{L} \left(\frac{-\beta_j}{A_j}\right)^m \prod_{j=1}^{m_j} (-1)^{n_j} b_{jv} \times \sum_{q=0}^{m_j-1} \frac{1}{q!} \left(\frac{m_{i\lambda}}{N_0}\right)^q \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{m_{i\lambda}}{N_0}\right)^n \times U\left(v; v + 1 - q - n; \frac{-\beta_j}{A_j}\right).
\]

**REFERENCES**


